



Using Mathematics Sample Items in the Classroom

Mathematics Assessment Office



Agenda

- ▶ Purpose and Research
- ▶ Classroom examples using sample items and released items
- ▶ Resources
- ▶ Summary



About You

- ▶ We'd like to know a little about who is out there.



Purpose

- ▶ **Kinds and Purposes of Assessment: Need a balanced system which includes: Summative, Interim, Formative, Classroom-based**
 - ▶ **Summative Assessments such as MSP and EOC**
 - ▶ **Purpose-Assessment of Learning**
 - System check; accountability
 - Increase overall standards
 - ▶ **Limitations**
 - Not detailed for individual diagnosis; not timely for individual
 - ▶ **Formative Assessment**
 - ▶ **Purpose-Assessment for Learning**
 - ▶ **Definition: “..planned process in which .. evidence of students’ status is used by teachers to adjust ...instructional procedures or by students to adjust their current learning tactics”**
 - ▶ **Limitations**
 - Challenging for teachers in deciding what, when, how often, for whom



Research

- ▶ *Envisioning a State Educational System: Improving Learning Through A Comprehensive Assessment System*, Joseph M. Ryan, Arizona State University, January 2010.
- ▶ *Effective Strategies for Teaching Students with Difficulties in Mathematics, Research Brief*, National Council of Teachers of Mathematics, 2007.



More Research

- ▶ http://thinkmath.edc.org/index.php/CCSS_Mathematical_Practices

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- NEW! Article on Symmetry and Line of Symmetry
- Common Core State Standards Mathematical Practices for Elementary School
- NCTM 2014 Indianapolis PowerPoint presentations



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CCSS Mathematical Practices

To emphasize the Mathematical Practices, the CCSS gives them their own distinct section, but they are not to be thought of as a separate supplements. The intent is that these *essential mathematical habits of mind and action* pervade the curriculum and pedagogy of mathematics. This document interprets and illustrates each of the eight Mathematical Practices as they might be exemplified in grades K–5.

Each article begins with the title of a Mathematical Practice, as given in the CCSS, followed by part or all of the CCSS description of that original text is an elementary school example that needs no further comment or is explicitly about content that goes beyond elementary school (...).

Click below for explanations and illustrations of each standard for elementary school.

1. [Make sense of problems and persevere in solving them](#)
2. [Reason abstractly and quantitatively](#)
3. [Construct viable arguments and critique the reasoning of others](#)
4. [Model with mathematics](#)
5. [Use appropriate tools strategically](#)
6. [Attend to precision](#)
7. [Look for and make use of structure](#)
8. [Look for and express regularity in repeated reasoning](#)

[Differences between, and connections between, Content and Practice standards](#)



Why Use Sample Items and Released Items for Formative Assessment Purposes

- ▶ Items/tasks are focused on learning goals and performance expectations
- ▶ Items give students familiarity with format of summative assessment
- ▶ Ideas can be adapted to any grade level
- ▶ Examples of classroom use provide advantages of formative assessment and best practices to increase learning.
 - ▶ Data provided directly to teachers and students



Using Multiple-Choice Items



Sam left Olympia at 4:30 pm traveling in a car and driving at the speed of 60 mph. What time will he reach his destination north of Seattle, which is 67 miles away?

- A. You're lying. No one drives 60 mph during rush hour.
- B. No less than two hours later, with a headache and white knuckles.
- C. All of the above.

Information about Multiple-Choice Items

- ▶ Answer choice distractors are purposefully written to include common misconceptions and common errors, which may be used for diagnostic purposes.



Examples of How to Use Multiple-Choice Items As Formative Assessments

- ▶ Have students construct answer before seeing answer choices.
- ▶ Have students explain the “attractiveness” of wrong answers to uncover common misconceptions.
- ▶ Have students justify the correct answer choice to ensure they didn’t get the right answer for the wrong reason.
- ▶ Have students determine the question, given the answer choices and answer.
- ▶ Have students make up another item with answer choices for classmates to do. This involves synthesis, which is a higher cognitive demand than identification of correct answer.



Sample item for performance expectation 3.3.A

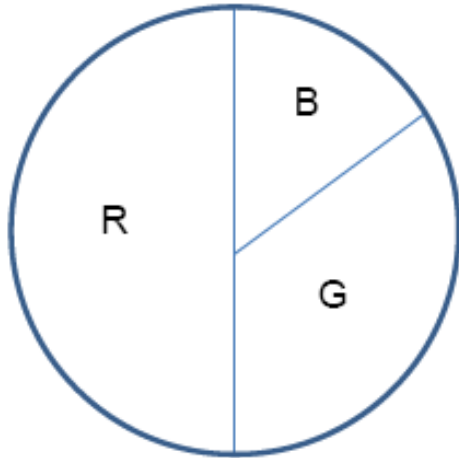
Jakob drew a circle. He colored the circle $\frac{1}{4}$ green (G), $\frac{1}{8}$ blue (B) and the rest of the circle red (R).

Which circle could represent the circle Jakob drew?

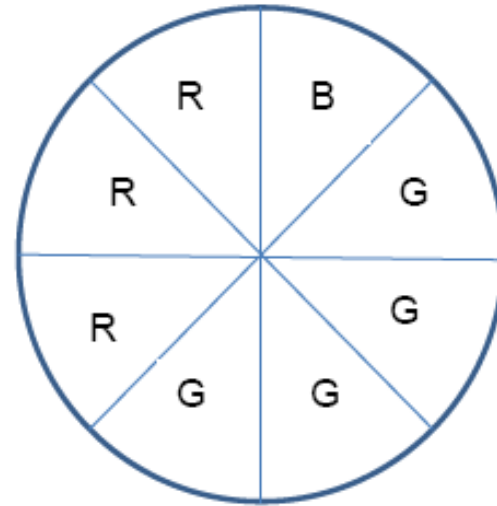
- ▶ Please draw a circle that could represent the one that Jakob drew.
- ▶ Share your circle with your neighbor and explain why your circle represents the circle Jakob drew.
- ▶ Compare your circle with your neighbor's circle. How are they alike and how are they different?



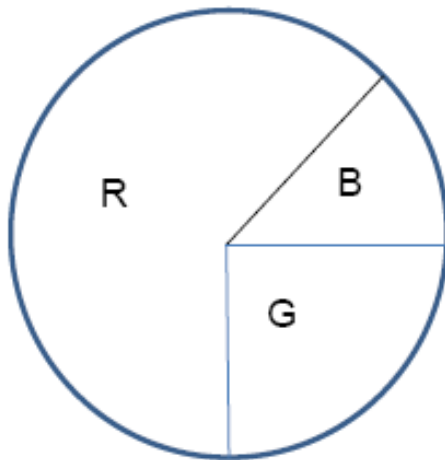
o A.



o C.



o B.



Which circle could represent the circle Jakob drew? How do you know?

Explain why someone might make a mistake and choose the wrong circles.

Sample item for Performance Expectation 6.4.C

Kevin is using chalk to draw a large circle on the school playground. The circumference of the circle is approximately 44 feet.

What is the approximate area of the circle?

- A. 154 square feet
- B. 308 square feet
- C. 616 square feet
- D. 1,078 square feet

- ▶ Have students construct answer before seeing answer choices.
- ▶ Have students analyze wrong answer choices together.
 - ▶ B. $A = 2\pi r^2$ C. $A = \pi d^2$ D. $A = 22r^2$ (instead of $22/7$)



Sample item for Performance Expectation A1.4.B/M1.3.D

Which equation represents the line that passes through the points (2, 2) and (4, 1)?

A. $y = -2x + 6$

B. $y = -\frac{1}{2}x + 3$

C. $y = \frac{1}{2}x + 1$

D. $y = 2x - 2$

- ▶ Have students construct answer before seeing answer choices.
- ▶ Have students analyze wrong answer choices



Henry is trying to find a shape.

He was given these clues:

- Opposite sides are equal.
- Opposite sides are parallel.
- The shape is a rectangle.

Which figure fits all the clues?

A.



B.



C.



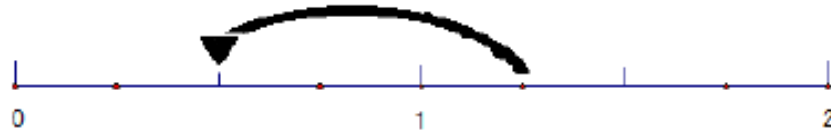
Another way to analyze
answer choices:

Clues that fit

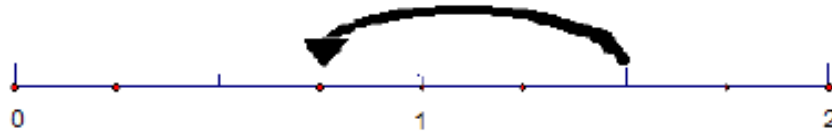
Clues that don't fit
and why.

Here are the answer choices and the answer, what is the question?

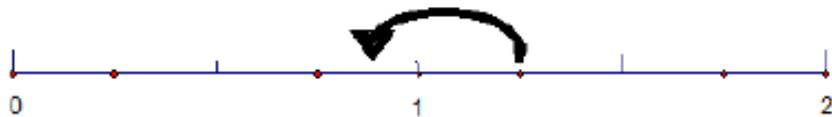
A.



B.



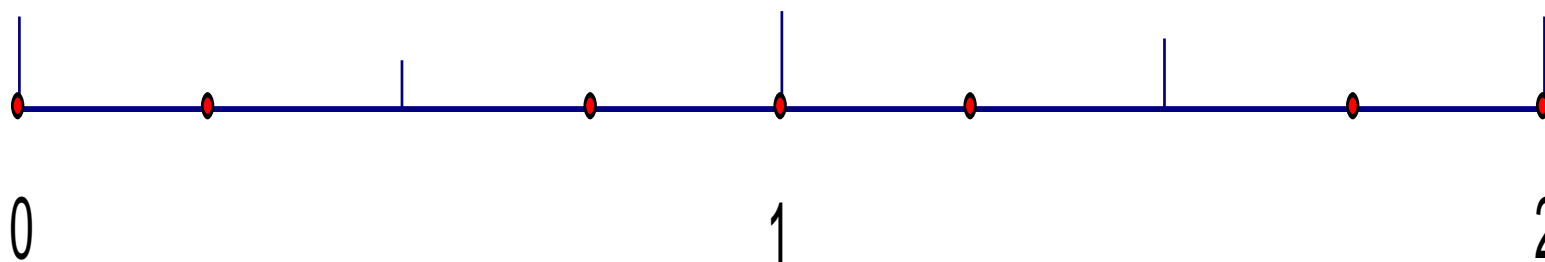
C.



Answer: C



Extension: Take blank number line and ask students to represent their own expressions or equations.



Examples of How to Use Multiple-Choice Items As Formative Assessments

- ▶ Have students construct answer before seeing answer choices.
- ▶ Have students explain the “attractiveness” of wrong answers to uncover common misconceptions.
- ▶ Have students justify the correct answer choice to ensure they didn’t get the right answer for the wrong reason.
- ▶ Have students determine the question, given the answer choices and answer.
- ▶ Have students make up another item with answer choices for classmates to do. This involves synthesis, which is a higher cognitive demand than identification of correct answer.



Using Short-Answer Items to develop understanding and promote vocabulary



Research

- ▶ *Giving Voice to English Language Learners in Mathematics*, Rafaela M. Santa Cruz; NCTM News Bulletin (January/February 2009)
<http://www.nctm.org/news/content.aspx?id=16895>

“Classroom instruction should support bilingual students by engaging them in mathematics conversations that go beyond the translation of vocabulary to include authentic communication about mathematical concepts. One of the goals of mathematics instruction for bilingual students should be to support the participation of all students, regardless of their proficiency in English, in discussions about mathematical ideas.”



More Research on Developing Mathematical Vocabulary

- ▶ http://thinkmath.edc.org/index.php/Developing_mathematical?vocabulary

Young children acquire vocabulary at an astonishing rate—a full 50% of what will be their adult vocabulary by the age of five! They do that *entirely from use in context*. **New words are acquired extremely rarely from a definition and never solely from one...but even adults learn new words and meanings mostly from context and usage *first*.**”



More Research on Developing Mathematical Vocabulary

- ▶ <http://www.nctm.org/news/content.aspx?id=16895>
- ▶ *Giving Voice to English Language Learners in Mathematics*,
Rafaela M. Santa Cruz, San Diego State University, NCTM
News Bulletin (January/February 2009)



Using Short-answer Items to Develop Mathematical Vocabulary through Usage

Karin and Felix were asked to describe these quadrilaterals.

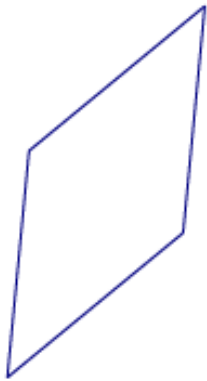


Figure A

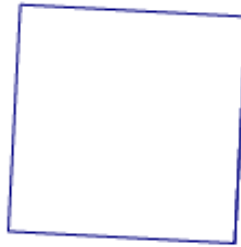


Figure B

Karin described the quadrilaterals in these ways:

- Figure A is a rhombus
- Figure B is not a rhombus because it is a square.

Are Karin's descriptions correct? Explain why or why not.

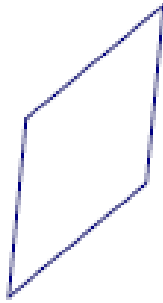


Figure A

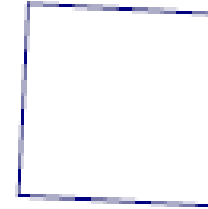


Figure B

Task:

- ▶ Explain why this statement is correct or not correct:
Figure A is a rhombus.
- ▶ Explain why this statement is correct or not correct:
Figure B is not a rhombus because it is a square.

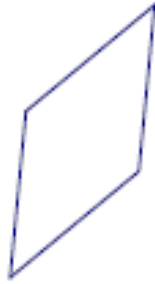


Figure A

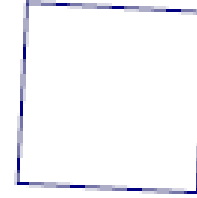


Figure B

- ▶ Explain why this statement is correct or not correct: Figure A is a rhombus.

- ▶ Vocabulary from prompt:

- ▶ correct or not correct; Figure A; rhombus; why → because

- ▶ Vocabulary from content:

- ▶ attributes of a rhombus: equal side lengths

- ▶ Explain why this statement is correct or not correct: Figure B is not a rhombus because it is a square.

- ▶ Vocabulary from prompt:

- ▶ correct or not correct; Figure A; rhombus; square; why → because

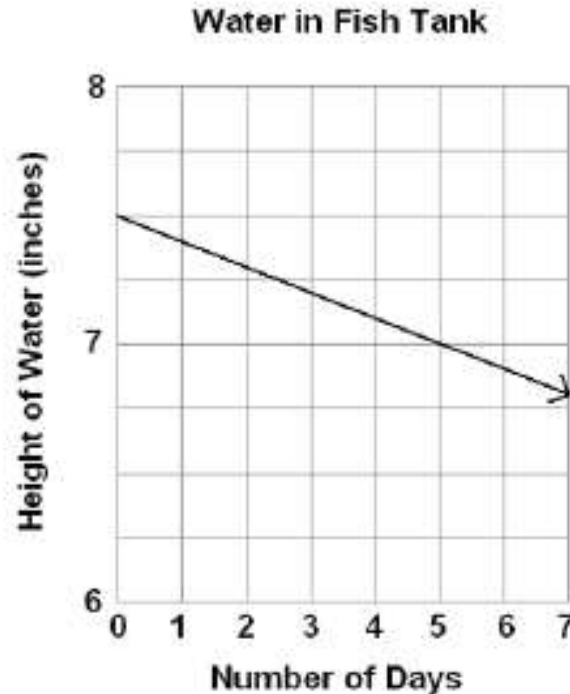
- ▶ Vocabulary from content:

- ▶ attributes of a rhombus and square; equal side lengths, right angles



Sample item for Performance Expectation 8.1.E

Dimitri's family has a fish tank. The height of the water in the fish tank during one week is represented by the line on the graph.



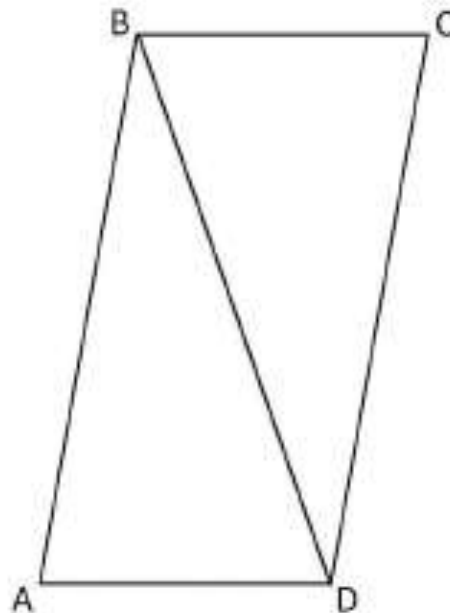
- Describe what the y -intercept of the line represents in the situation.
- Describe what the slope of the line represents in the situation.



Sample item for performance expectation G.3.B/M2.3.F

In the diagram:

- Quadrilateral ABCD with $\overline{AB} \cong \overline{DC}$
- $\angle ABD \cong \angle CDB$



Prove $\triangle ABD \cong \triangle CDB$ using mathematical language and concepts.



Tech to Assess Communication

- ▶ Factor the polynomial $16x^4 - 81$
 - ▶ Student's original answer: $(4x^2 + 9)(4x^2 - 9)$
 - ▶ Student's corrected answer: $(4x^2 + 9)(2x + 3)(2x - 3)$
- ▶ Factor the polynomial $y^3 - 1$
 - ▶ Student's original answer: $(y - 1)^3$
 - ▶ Student's corrected answer: $(y - 1)(y^2 + y + 1)$
- ▶ Student mathcasts



Mathcast Resources

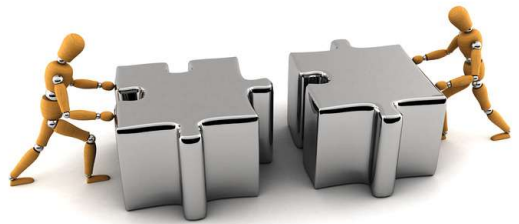
- ▶ Math247: <http://math247.pbworks.com/>
- ▶ Mathcasts – Digital Learning Dept @ OSPI:
<http://digitalllearning.k12.wa.us/resources/teaching/mathcasts.php>
- ▶ Mathcasts @ Patty Papers:
<http://pattyoflynn.edublogs.org/mathcasts/>



Strategies & Tools

Strategies

- ▶ Matching



- ▶ Ordering



Tools

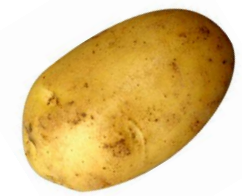
- ▶ Note cards



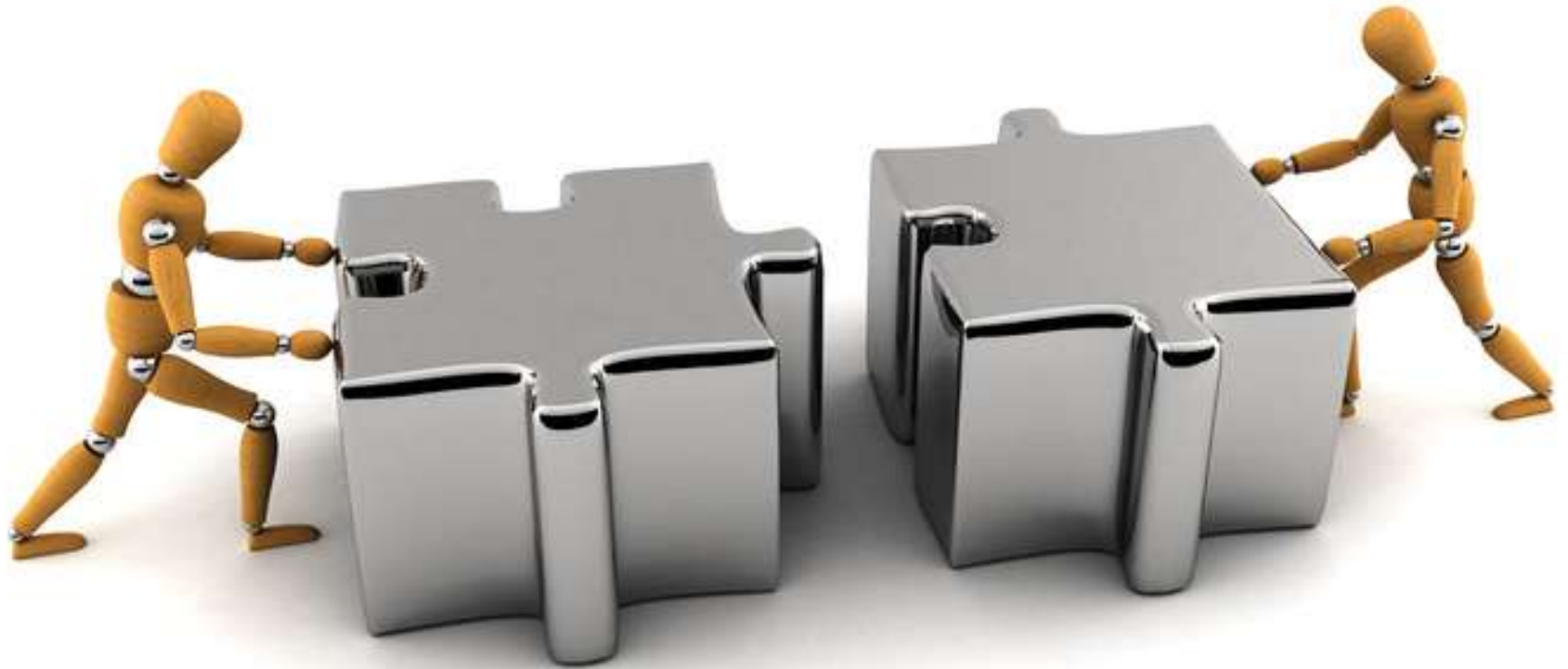
- ▶ Bodies



- ▶ Hot Potatoes



Matching



Content Ideas for Matching

▶ Equivalent expressions

- ▶ Evaluating or simplifying expressions
- ▶ Operations/arithmetic
- ▶ Fractions, decimals, percents, scientific notation

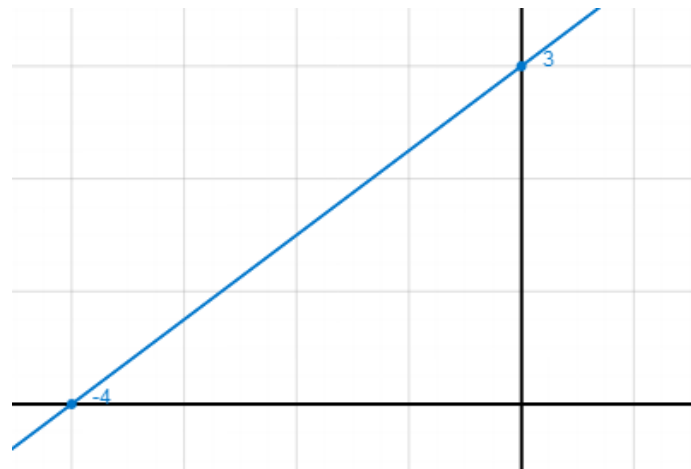
$$\left(\frac{x^2}{x^{-3}}\right)^4 = x^{20} \quad \sqrt{\frac{18}{25}} = \frac{3\sqrt{2}}{5} \text{ or } \frac{3}{5}\sqrt{2}$$



Content Ideas for Matching

- ▶ **Equivalent expressions**
 - ▶ Evaluating or simplifying expressions
 - ▶ Operations/arithmetic
 - ▶ Fractions, decimals, percents, scientific notation
- ▶ **Equations/inequalities and their graphs**

$$y = \frac{3}{4}x + 3$$



Content Ideas for Matching

- ▶ **Equivalent expressions**
 - ▶ Evaluating or simplifying expressions
 - ▶ Operations/arithmetic
 - ▶ Fractions, decimals, percents, scientific notation
- ▶ **Equations/inequalities and their graphs**
- ▶ **Factoring polynomials**

$$|6x^2 - 8|$$

$$(4x + 9)(4x - 9)$$



Content Ideas for Matching

- ▶ **Equivalent expressions**
 - ▶ Evaluating or simplifying expressions
 - ▶ Operations/arithmetic
 - ▶ Fractions, decimals, percents, scientific notation
- ▶ **Equations/inequalities and their graphs**
- ▶ **Factoring polynomials**
- ▶ **Terms and definitions or diagrams**

A location
in space
is a...

POINT



Content Ideas for Matching

- ▶ Equivalent expressions
 - ▶ Evaluating or simplifying expressions
 - ▶ Operations/arithmetic
 - ▶ Fractions, decimals, percents, scientific notation
- ▶ Equations/inequalities and their graphs
- ▶ Factoring polynomials
- ▶ Terms and definitions or diagrams
- ▶ Perpendicular or parallel lines

$$y = \frac{3}{4}x + 3$$
$$3x - 4y = 6$$



Ideas for Use

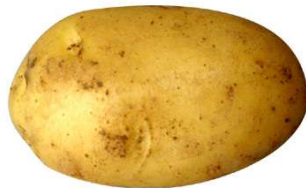
Tools

▶ Note cards

- ▶ I have... Who has...



▶ Hot Potatoes



▶ Bodies

- ▶ Mix-n-match



Matching

- ▶ **Sample item from Grades 6-8 Updates for 2012, page 25:**

Determine the value of the expression.

$$-|4 - 13| + 5$$

What is the value of the expression?

- A. 14
- B. 4
- C. -4
- D. -12



Ideas for Classroom Use

- ▶ I have... / Who has...



Matching

- ▶ **Sample items from EOC Updates for 2011, page 14:**

Write the expression in simplest radical form.

$$\sqrt{\frac{18}{25}}$$

Simplify the expression.

$$\left(\frac{x^2}{x^{-3}}\right)^4$$



Ideas for Classroom Use

▶ Mix-n-match

$$\left(\frac{x^2}{x^{-3}}\right)^4 = x^{20}$$



$$\sqrt{\frac{18}{25}} = \frac{3\sqrt{2}}{5} \text{ or } \frac{3}{5}\sqrt{2}$$



Matching

► **From EOC Updates for 2012, pg 14:**

Sample item for Performance Expectation A1.2.A/M1.6.A

Which numbers are both less than $-\frac{5}{6}$?

O A. -2.1 and $-\frac{6}{5}$

O B. $-\frac{2}{3}$ and $-\frac{3}{4}$

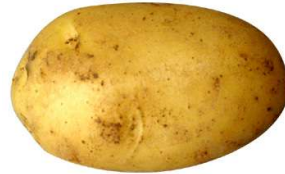
O C. -0.65 and -1.2

O D. $-\frac{2}{3}$ and -0.8



Ideas for classroom use

▶ Hot Potatoes



-1.1

$-1\frac{1}{10}$

$-\frac{3}{4}$

-0.75

-0.8

$-\frac{4}{5}$

$-\frac{5}{6}$

-0.8333...

$-\frac{2}{3}$

-0.666...

-2.1

$-2\frac{1}{10}$

$-\frac{6}{5}$

-1.2

-0.65

$-\frac{13}{20}$



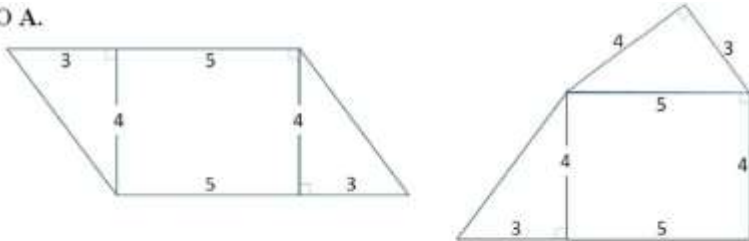
Hot Potatoes Cloze Activity

► EOC Updates for 2012, pg 30

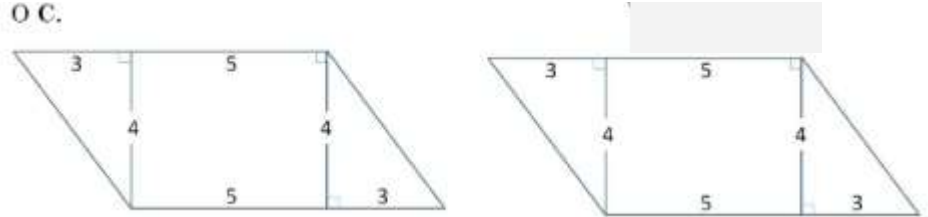
Christine knows that if two polygons are congruent, then they must have the same perimeter and area. She concludes that it is also true that if two polygons have the same perimeter and area, then they are congruent.

Which pair of polygons can be used as a counterexample to Christine's conclusion?

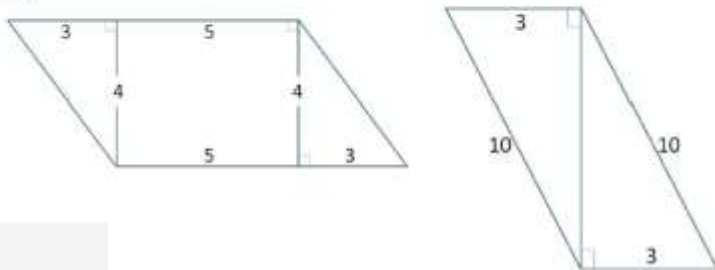
O A.



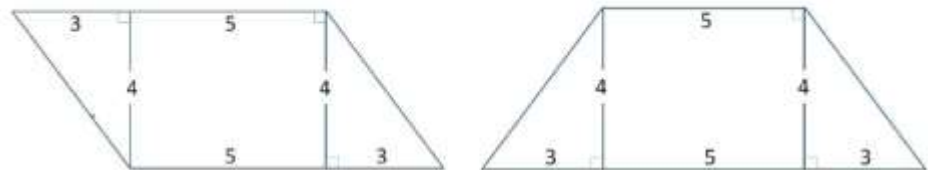
O C.



O B.

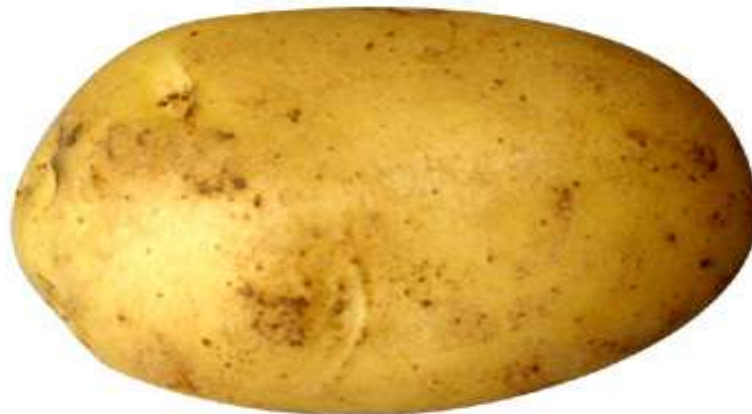


O D.



Hot Potatoes Cloze Activity

- ▶ EOC Updates for 2012, pg 30
- ▶ Cloze activity
 - ▶ http://moodle.ospi.k12.wa.us/file.php/18/hotpotatoes/Updates2012_pg30_cloze.htm



Trigonometry Properties

Pythagorean Properties

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \cot^2 x = \csc^2 x \quad 1 + \tan^2 x = \sec^2 x$$

Reciprocal Properties

$$\sin x \csc x = 1 \quad \cos x \sec x = 1 \quad \tan x \cot x = 1$$

Quotient Properties

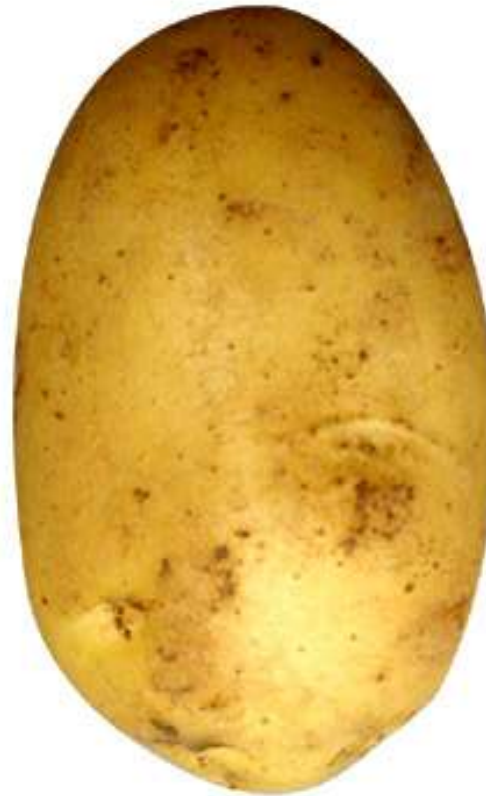
$$\tan x = \frac{\sin x}{\cos x} = \frac{\sec x}{\csc x}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{\csc x}{\sec x}$$



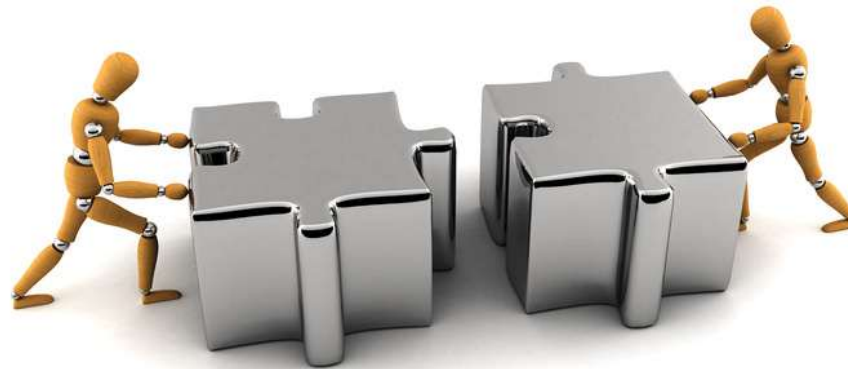
Trigonometry Properties

- ▶ [Flash cards](#)
- ▶ [Drag and Drop quiz](#)



Other Matching Ideas

- ▶ Matching puzzles
 - ▶ [NCTM Illuminations](#)
 - ▶ [Trigonometry Square #1](#): Matching values
 - ▶ [Trigonometry Square #2](#): Matching expressions



Ordering



Content Ideas for Ordering

► Numbers

$$\begin{array}{ccc} & 2.7 & \sqrt{6.25} \\ -2\frac{3}{4} & & \\ & 2.75 \times 10^{-1} & \end{array}$$



Content Ideas for Ordering

- ▶ Numbers
- ▶ Expressions

$$f(-1)$$

for the function

$$f(x) = 3 - 2x^2$$

$$a_7$$

if $a_1 = 8$ and

$$a_{k+1} = a_k + 3$$

$$4y - 3|x - z^2|$$

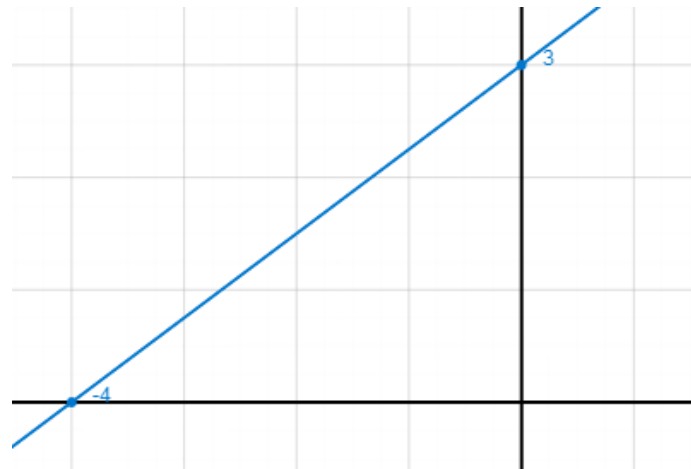
where $x = 2, y = 5$
and $z = -3$



Content Ideas for Ordering

- ▶ Numbers
- ▶ Expressions
- ▶ Lines by slope

$$y = \frac{3}{2}x - 4$$



The line through points
(1, 2) and (3, 4)



Content Ideas for Ordering

- ▶ Numbers
- ▶ Expressions
- ▶ Lines by slope
- ▶ Equations

$$2^x = 16^5$$

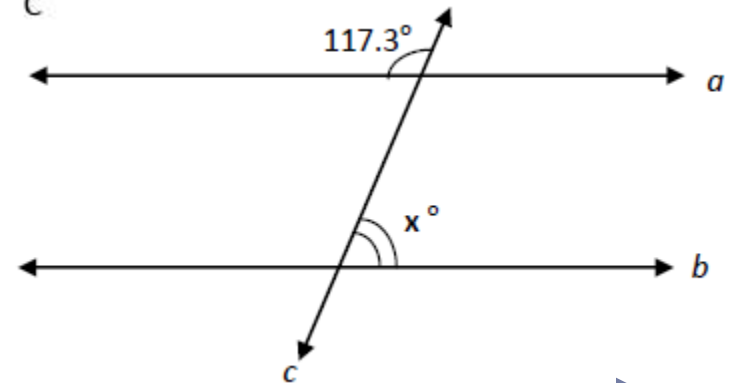
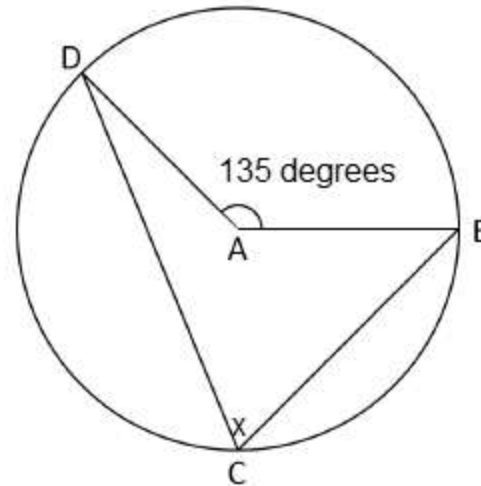
$$\begin{cases} 3x - 2y = 6 \\ x - y = 1 \end{cases}$$

$$x^2 + 10x - 24 = 0$$



Content Ideas for Ordering

- ▶ Numbers
- ▶ Expressions
- ▶ Lines by slope
- ▶ Equations
- ▶ Angle measures



Content Ideas for Ordering

Sample item for Performance Expectation G.3.B/M2.3.F

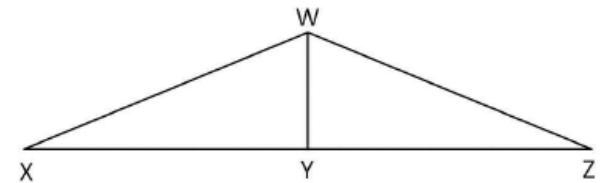
- ▶ Numbers
- ▶ Expressions
- ▶ Lines by slope
- ▶ Equations
- ▶ Angle measures
- ▶ Proofs

A proof is shown.

Fill in the blanks for steps 4 and 5 to complete the proof.

Given: \overline{WY} is the perpendicular bisector of \overline{XZ}

Prove: $\triangle WXY \cong \triangle WZY$



Statements	Reasons
1. \overline{WY} is the perpendicular bisector of \overline{XZ} .	1. Given
2. $\angle WYX \cong \angle WYZ$	2. Perpendicular lines form 90 degree angles
3. $\overline{WY} \cong \overline{WY}$	3. Reflexive property of congruence
4.	4. A bisector divides a segment into two equal halves
5. $\triangle WXY \cong \triangle WZY$	5.



Ordering

► **From EOC Updates for 2012, pg 14:**

Sample item for Performance Expectation A1.2.A/M1.6.A

Which numbers are both less than $-\frac{5}{6}$?

O A. -2.1 and $-\frac{6}{5}$

O B. $-\frac{2}{3}$ and $-\frac{3}{4}$

O C. -0.65 and -1.2

O D. $-\frac{2}{3}$ and -0.8



Ideas for classroom use

Tools

▶ Note cards

-1.1

-3/4

-0.8

-5/6

-2/3

-2.1

-6/5

-0.65



Ideas for classroom use

Tools

- ▶ Note cards
- ▶ Bodies
- ▶ Line-ups



Line-ups



Ideas for classroom use

Tools

- ▶ Note cards
- ▶ Bodies
 - ▶ Line-ups
- ▶ Hot Potatoes
 - ▶ Drag & drop quiz



Ordering

► From EOC Updates for 2012, pg 42:

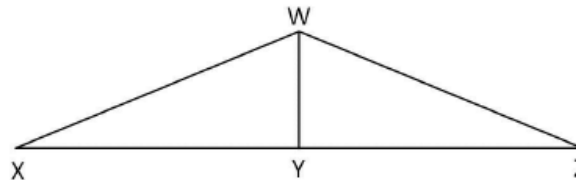
Sample item for Performance Expectation G.3.B/M2.3.F

A proof is shown.

Fill in the blanks for steps 4 and 5 to complete the proof.

Given: \overline{WY} is the perpendicular bisector of \overline{XZ}

Prove: $\triangle WXY \cong \triangle WZY$



Statements	Reasons
1. \overline{WY} is the perpendicular bisector of \overline{XZ} .	1. Given
2. $\angle WYX \cong \angle WYZ$	2. Perpendicular lines form 90 degree angles
3. $\overline{WY} \cong \overline{WY}$	3. Reflexive property of congruence
4.	4. A bisector divides a segment into two equal halves
5. $\triangle WXY \cong \triangle WZY$	5.



Ideas for classroom use

Tools

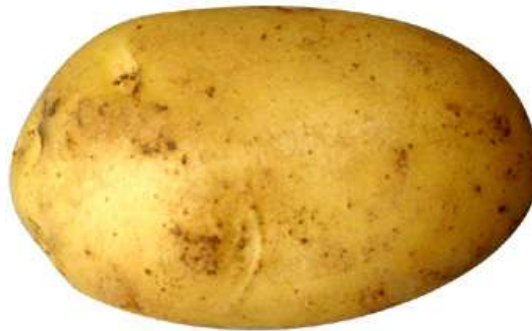
- ▶ Note cards



Ideas for classroom use

Tools

- ▶ Note cards
- ▶ Hot Potatoes
 - ▶ http://moodle.ospi.k12.wa.us/file.php/18/hotpotatoes/Updates2012_pg42_drag_drop.htm



Hot Potatoes

- ▶ <http://hotpot.uvic.ca/>
- ▶ Freeware
- ▶ Creates interactive activities
 - ▶ multiple-choice
 - ▶ short-answer
 - ▶ crossword
 - ▶ matching/ordering (drag and drop)
 - ▶ flashcards
 - ▶ gap-fill (cloze) exercises

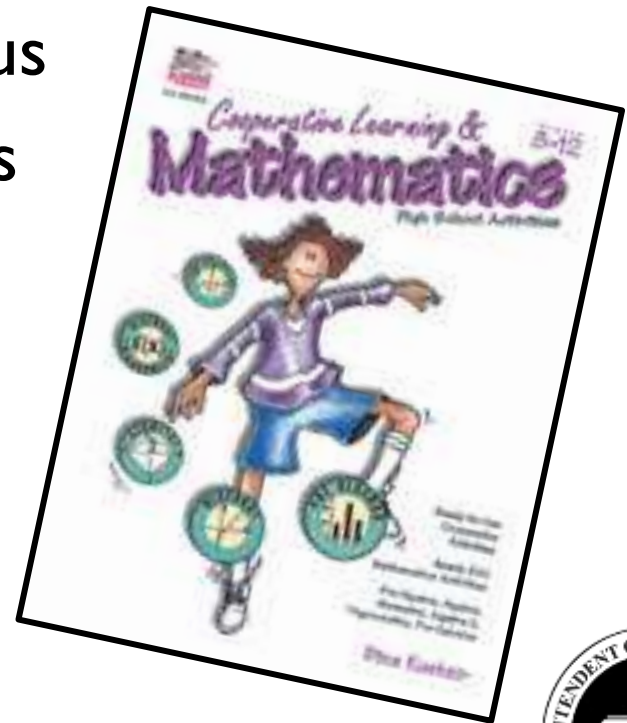


Hot Potatoes™
From Half-Baked Software Inc



Line-ups and Mix-N-Match

- ▶ Cooperative Learning & Mathematics by Dina Kushnir
- ▶ <http://www.kaganonline.com/catalog/mathematics.php>
- ▶ Pre-Algebra through Pre Calculus
- ▶ Cooperative Learning structures
 - ▶ Line-Ups
 - ▶ Mix Pair RallyCoach
 - ▶ Mix-N-Match
 - ▶ Inside-Outside Circle
 - ▶ RallyCoach
 - ▶ RoundTable



Strategies

- ▶ **Sorting**
- ▶ **Asking questions**
 - ▶ Generating questions
 - ▶ “If I knew...”
- ▶ **Scoring student responses**
- ▶ **Peer review**



Sorting

► **From page 15 of 3-5 document:**

Henry is trying to find a shape.

He was given these clues:

- Opposite sides are equal.
- Opposite sides are parallel.
- The shape is a rectangle.

Which figure fits all the clues?



Ideas for Use

- ▶ Match Property to Figure
- ▶ Venn Diagram
- ▶ Can be done using:
 - ▶ Smartboard
 - ▶ Notecards



Sorting

▶ **From page 58 of 6-8 document:**

Nadifa is designing an art project. She will use pieces of wire for the outline of each shape in the art project. She wants one shape to be a right triangle. She has these lengths of wire.

3 inches, 5 inches, 8 inches, 12 inches, 13 inches



Ideas for Use

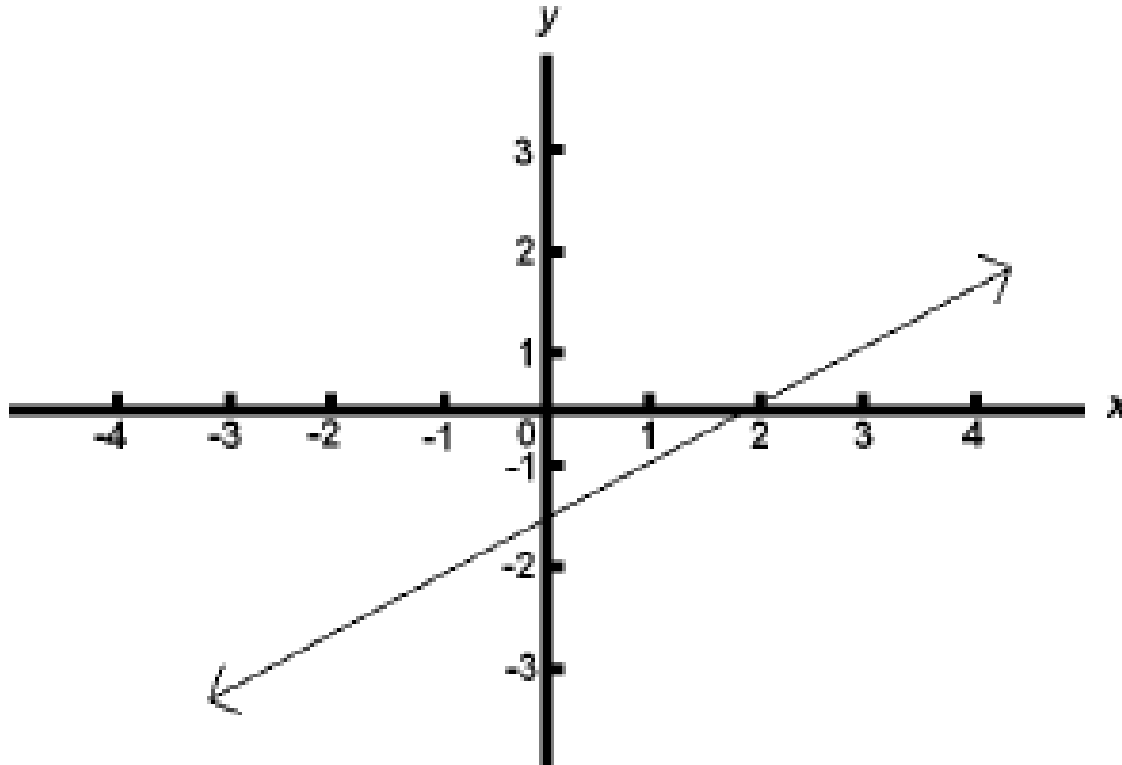
- ▶ Given groups of three numbers:
 - ▶ Grade 5:
 - ▶ Triangles with the same perimeter
 - ▶ Grade 8:
 - ▶ Right triangle versus non-right triangles
 - ▶ EOC:
 - ▶ Triangle possible versus not possible
 - ▶ Acute, right or obtuse triangle



Sorting

► **From page 47 of 6-8 document:**

The graph represents a linear function.



Ideas for use

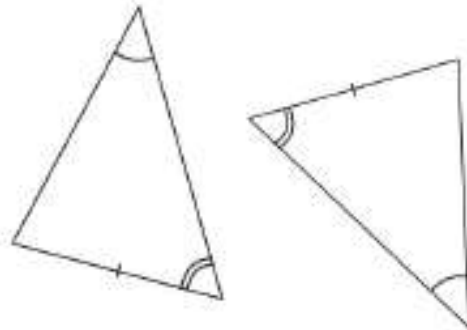
- ▶ Match Graph to Description



Sorting

► From page 31 of EOC document:

The triangles shown are congruent.



Which postulate or theorem could be used to conclude the triangles are congruent?

AAS

ASA

SAS

SSS



Ideas for Use

- ▶ Sort pairs of triangles by the congruence statement(s) that could be used to prove congruence
- ▶ Pairs of triangles on cards that are placed into categories such as ASA, SSS, AAS, and SAS



Ideas for Use

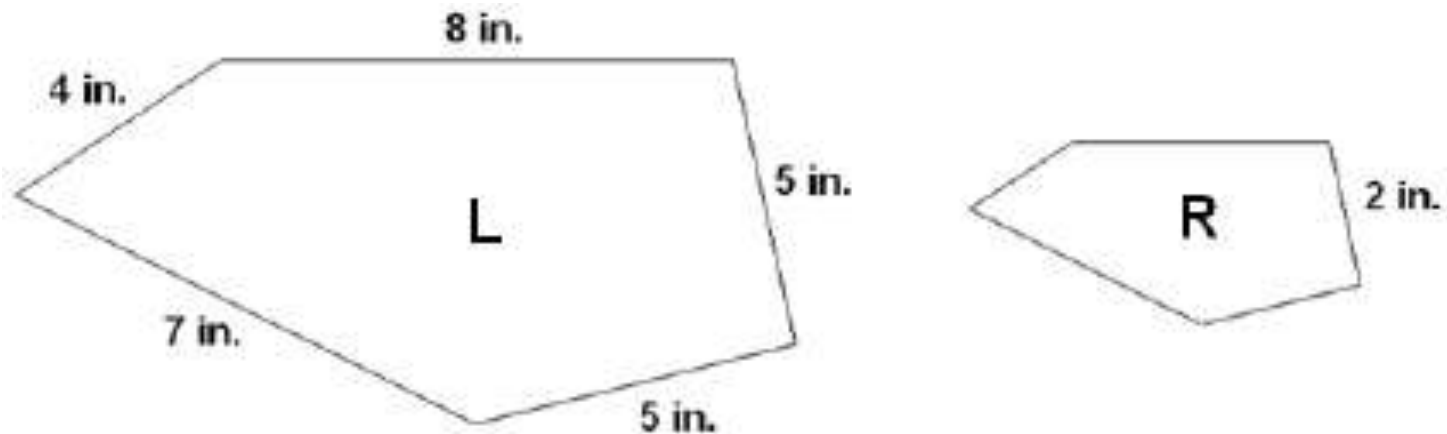
- ▶ Given two “pieces” that correspond on two triangles, such as a side and an angle:
 - ▶ Identify all the congruence statements that could be used and what information would be needed to prove that particular statement
 - ▶ Congruent Triangles



Generating Questions

► **From page 40 of 6-8 document:**

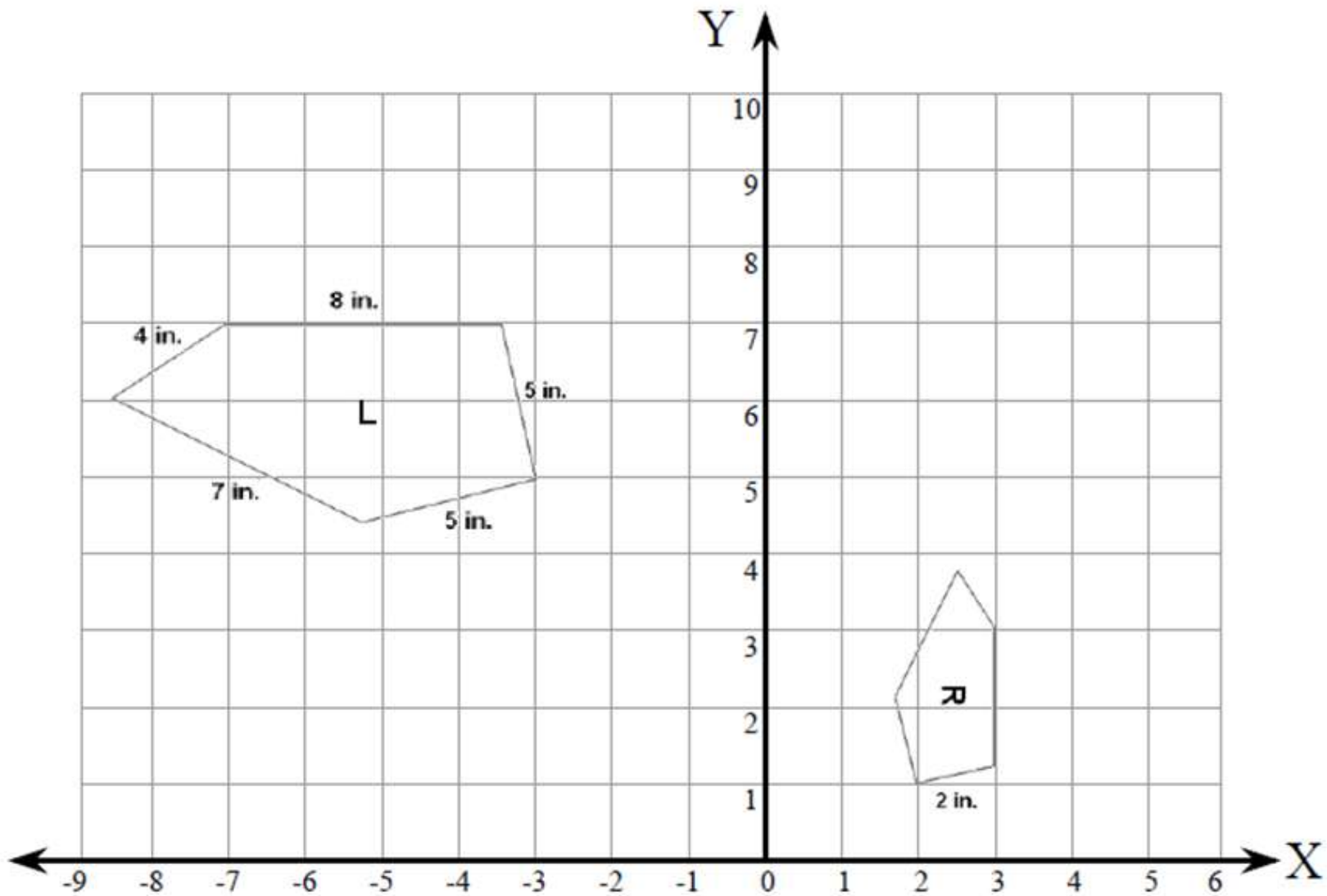
Pentagon L and pentagon R are similar.



Other Questions

- ▶ What is the scale factor between pentagon L and pentagon R?
- ▶ Put figures on a coordinate plane
 - ▶ Describe a combination of rotations, reflections, translations, and/or dilations that could be used to transform pentagon L onto pentagon R
 - ▶ Have students challenge each other to describe the transformations needed to superimpose one figure onto the other





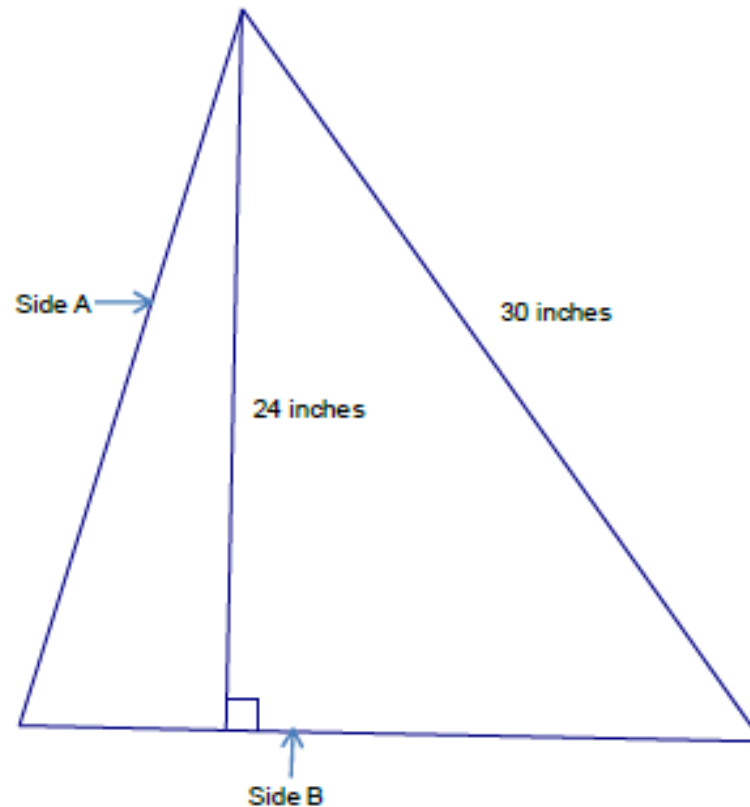
If I knew...

- ▶ Pentagon R and pentagon L are machine parts. The cost to produce a part is proportional to its area. It costs \$36 to produce a part shaped like pentagon R.
- ▶ Ask: What is the cost to produce a part shaped like pentagon L?



Generating Questions

- ▶ **From page 56 of 3-5 document:**



If I knew...

- ▶ The length of side B is 25 inches.
 - ▶ Ask: What is the area of the triangle?
- ▶ The area of the triangle is 300 square inches.
 - ▶ Ask: Can you determine the length of Side A? Can you determine the length of Side B? Explain.
- ▶ Side A and Side B have the same length. The measure of angle a is 53° .
 - ▶ Ask: What is the perimeter of the triangle to the nearest inch.



Generating Questions

► **From page 20 of 3-5 document:**

Animals at the zoo were weighed.

Animal	Weight in Pounds
Elephant	7,243
Hippopotamus	5,319
Walrus	3,209
Grizzly Bear	1,028
Rhinoceros	3,869
Orca Whale	8,003



Other Questions

- ▶ What is the mean/median weight of the animals?
- ▶ Approximately how many grizzly bears would it take to weigh the same as 1 hippopotamus?
- ▶ Which animal weighs the closest to 2 tons?
- ▶ Which animal weighs the closest to 1,450 kilograms? (Would give a conversion factor such as **1 pound \approx 0.45 kilograms**)



If I knew...

- ▶ There are 2 Walruses at the zoo. Their combined weight is 6,115 pounds.
- ▶ Ask: What is the weight of the second walrus?



If I knew...

- ▶ The typical weight of an adult Orca Whale is 9,000 pounds.
- ▶ Ask: What percent of a typical adult Orca Whale's weight is the weight of the Orca Whale at the zoo?
- ▶ Ask: What percent of the Orca Whale's current weight will it need to gain to weigh 9,000 pounds.



Generating Questions

► **From page 34 of 3-5 document:**

Julie has a bag of 50 balls that are the same size and shape. The balls come in 4 different colors: red, white, green, and yellow.

In the bag:

- There are 6 red balls
- there are twice as many white balls as red balls
- there are 14 green balls
- the rest of the balls are yellow.



Other Questions

- ▶ How many yellow balls are in the bag?
- ▶ What is the probability of pulling a yellow ball?
- ▶ When Julie pulls one ball from the bag, does not replace it, and pulls a second ball from the bag, what is the probability both balls will be green?



If I knew...

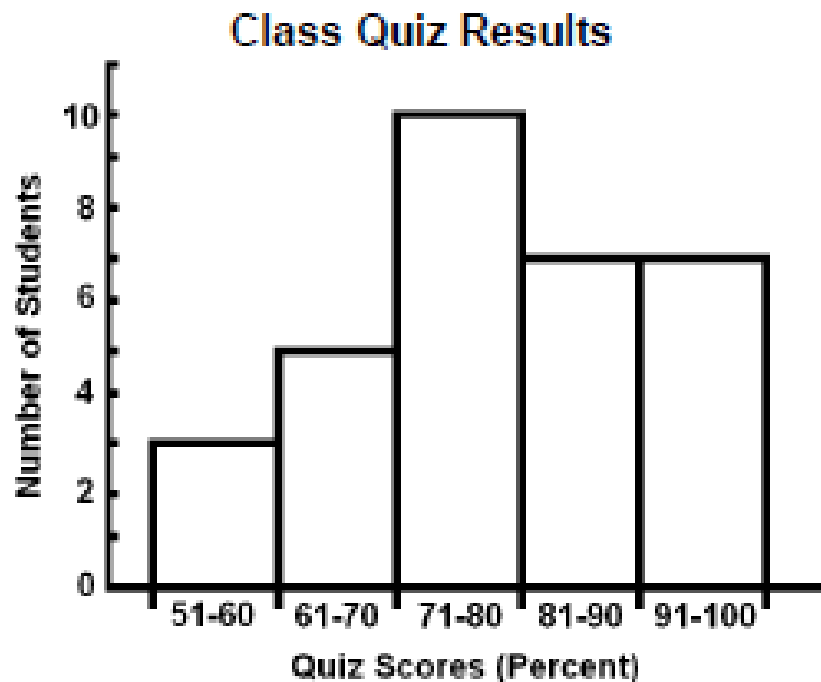
- ▶ Another bag has 125 balls, but the probability of pulling a red ball from this bag is the same as the probability of pulling a red ball from the bag with 50 balls.
- ▶ Ask: How many red balls are in the bag with 125 balls?



Generating Questions

► **From page 45 of 6-8 document:**

Javier's teacher created a histogram of the students' quiz scores. The histogram shows the quiz scores for 32 students in the class.



Other Questions

- ▶ How many more students scored between 71% and 80% than scored between 51% and 60%?
- ▶ What is the probability that a student, selected at random, scored between 71% and 80%?
- ▶ Create a circle graph that represents the same information as the histogram.



If I knew...

- ▶ There are 6 more students who need to take the quiz.
 - ▶ Ask: What is the fewest number of students who need to score 81% or greater to make the median test score 81% or greater?
- ▶ The scores that fall between 81% and 90% are 83%, 83%, 83%, 86%, 86%, 86%, and 89%.
 - ▶ Ask: What is the upper quartile of the quiz scores?



Generating Questions

▶ **From page 24 of EOC document:**

The admission fee at a small local fair was \$1.50 for children and \$4.00 for adults. A total of 2,200 people entered the fair and \$5,050 was collected.



If I knew...

- ▶ The fair committee determined that next year they need to collect \$6,000 in admission fees. From previous years' data, the committee estimates the attendance for next year will be approximately 5% greater than the attendance this year.
- ▶ Ask: Suggest an admission fee for children and adults that would allow the fair to collect \$6,000.



Scoring

- ▶ Released Item Quick Guide Documents

<http://www.k12.wa.us/Mathematics/ReleasedItems.aspx>

- ▶ Rubrics and scored, annotated student work



Suggestions for Use

- ▶ **Start with previous year's or course's items**
 - ▶ Students can focus on the task of scoring
- ▶ **Have students:**
 - ▶ Work the problem
 - ▶ Discuss sample 2-point and 1-point responses
 - ▶ Discuss rubric
 - ▶ Score sample student work
 - ▶ Score their own work and then each other's



Released Item

▶ From page 39 of grade 4 Quick Guide

Mayumi walks around the school yard each day at recess. It takes her **5 minutes** to walk one time around the yard.

Mayumi starts walking at 1:30 P.M. and stops at 2:00 P.M.

How many times does Mayumi walk around the yard at recess?

Show how you got your answer using words or numbers.



Sample 2-point Response

$5 \text{ min} = 1 \text{ time}$ $10 \text{ min} = 2 \text{ times}$ $15 \text{ min} = 3 \text{ times}$ $20 \text{ min} = 4 \text{ times}$ $25 \text{ min} = 5 \text{ times}$ $30 \text{ min} = 6 \text{ times}$	$\begin{array}{r} 0:30 \\ + 0:30 \\ \hline 0:60 \end{array}$ $1:00 + 30 \text{ min.} = 1:30$ $1:30 + 30 \text{ min.} = 2:00$
I just counted by fives until I got to another 30 min. and added 30 plus 30 = 60 so and 60 min. is an hour but I only needed 30 mins. so I counted by fives again and each five mins. was one time and that gave me my answer.	
How many times does Mayumi walk around the yard at recess? <u>6 times</u>	



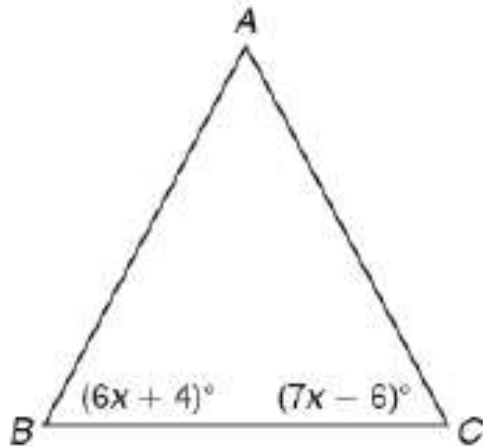
Rubric

- ▶ A 2-point response: The student shows understanding of using concepts and procedures from number sense and measurement by doing the following:
 - ▶ Shows use of 30 minutes or 5-minute intervals
 - ▶ Writes 6.
- ▶ A 1-point response: The student does one of the following:
 - ▶ Shows use of 30 minutes
 - ▶ Uses intervals of 5
 - ▶ Writes 6 or a number that is consistent with an error in the passage of time calculation.



Released Item

- ▶ **From page 17 of EOC Year 2 Quick Guide**
In the isosceles triangle shown, $AB = AC$.



What is the value of x ?

Support your answer using words, numbers, and/or diagrams.



Sample 2-point Response

$$6x + 4 = 9x - 6$$
$$+6 \quad -6x$$
$$\frac{10 = 1x}{1}$$

$x = \underline{10}$



Sample 1-point Response

$$\begin{aligned} & (6x + 4)(7x - 6) \\ & - 6x - 4 = -7x + 4 \\ & + \frac{7x}{} \\ & \quad \quad \quad \underline{1x = 10} \\ & \quad \quad \quad y = 10 \frac{5}{1} \quad x = 5 \\ \\ & \# = \underline{6} \end{aligned}$$



Rubric

- ▶ A 2-point response: The student applies concepts and procedures from geometric sense and algebraic sense by relating the equal sides of the triangle to their corresponding equal angles and writing and solving an equation or showing another valid method to find the value of x , which is 10.
- ▶ A 1-point response: The student does one of the following:
 - ▶ Shows algebraic work but never writes an equation, and the solution is correct
 - ▶ Writes a correct or mostly correct equation, but the solution is incorrect or missing
 - ▶ Implies a correct equation, but the solution is incorrect
 - ▶ Gives a correct answer with no work or incorrect work is shown.



Extensions

- ▶ Discuss different ways to show understanding, e.g., lists, computation, verbal description
- ▶ Evaluate efficiency, organization, clarity of different methods of showing work
- ▶ Have students write rubrics to score their own and other's work



Peer Review

- ▶ **Structure**

- ▶ Assessment Training Institute

- <http://www.assessmentinst.com/>

- ▶ Rick Stiggins

- <http://www.assessmentinst.com/author/rick-stiggins/>

- ▶ **Modeled and Practiced**



Peer Review

- ▶ **Feedback should be:**
 - ▶ **Specific:** Students should avoid ambiguity such as “it,” “big,” and “close to”
 - ▶ **Descriptive:** Encourage students to use verbal and written mathematical vocabulary
 - ▶ **Clarified:** Encourage students to ask probing questions such as “what did you do to go from this step to that step?”



Peer Review

▶ **From page 28 of EOC document:**

There are 17 employees at a company. The employees earn these hourly wages.

— 8 employees earn \$9 per hour.

— 7 employees earn \$12 per hour.

— 2 employees earn \$33 per hour.

Determine and explain mathematically which measure of center best represents the hourly wages of employees at the company.



Peer Review

- ▶ “Best represents”
 - ▶ Has the response addressed “best” through comparison or evidence
 - ▶ There may be more than one measure that can be supported as “best”



Peer Review

► **From page 52 of 3-5 document:**

Look at the numbers in the chart.

Starting Number	New Number
1	4
3	6
5	8
6	9

Write a rule that describes how to go from the starting number to the new number.



Peer Review

- ▶ “Plus three to the starting number”
- ▶ “Add three to it”
- ▶ “The starting number plus three is the new number”
- ▶ “The new number is three bigger than the starting number”
- ▶ “It goes up by three each time”



Peer Review

► **From page 39 of 3-5 document:**

Anton gave his classmates clues for 2 rectangles he drew.

- Each rectangle has a perimeter of 30 inches.
- Rectangle A has an area of 50 square inches.
- Rectangle B has a different area than rectangle A.

Draw a rectangle that fits the clues for rectangle B.

Be sure to label the length and width of your rectangle.



Peer Review

- ▶ **Use a checklist**
 - ▶ Does the rectangle have all the required characteristics
- ▶ **Compare two or more correct answers**
 - ▶ Is one answer “easier” than others?
 - ▶ Is one answer “more likely” than others?
- ▶ **What answers were not considered?**
 - ▶ Fraction/decimal side lengths?
 - ▶ A rectangle that is also a square?



Peer Review

▶ **From page 22 of 6-8 document:**

Jerome is making folded paper cranes. In 1 hour he made 17 folded paper cranes. He knows it takes the same amount of time to make each folded paper crane.

Write an equation with one variable that Jerome could use to determine how much time it takes to make each folded paper crane.

Be sure to define the variable.



Peer Review

▶ Equations:

- ▶ Look for equivalent equations and discuss how they represent the situation in different ways

▶ Variable:

- ▶ Can vary because “time” could be in terms of hours, minutes, etc.
- ▶ Look for clarity in definition
 - ▶ “Time”
 - ▶ “Minutes”
 - ▶ “Number of minutes it takes to make one paper crane”



Peer Review

▶ **From page 40 of EOC document:**

The given statement is a valid geometric proposition.

If two angles are right angles, then they are congruent.

Write the converse of the given statement.

Determine whether the converse is valid or invalid and provide evidence (example or counterexample) for your answer.



Peer Review

▶ Examples and Counterexamples

- ▶ What conditions must an example meet?
- ▶ What conditions must a counterexample meet?
- ▶ When asking for contrapositive or inverse, what are equivalent statements?
 - ▶ Is “do not have the same measure” equivalent to “not congruent”
 - ▶ Is “an acute or obtuse angle” equivalent to “not a right angle”





Thank you!

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